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How to Use This Addenda

UNIT SYNOPSIS

In this unit, students build on their previous understanding of triangle similarity to explore special right triangles (45°-45°-90° and 30°-60°-90°). The goal is not necessarily that students memorize the ratios of each side, since that information is given on the reference sheet that they can always use; rather, what is important is that the ratios work because, by angle-angle similarity, all 45°-45°-90° triangles are similar to one another, as all 30°-60°-90° triangles are to each other. Students then explore the trigonometric ratios sine, cosine, and tangent, and their inverse. Again, the emphasis is on understanding that we can use these trigonometric ratios because triangles with the same angles must be similar. Sine, cosine, and tangent ratios represent the entire set of similar triangles. The unit concludes with trigonometry application problems including angles of elevation and depression.

CONTENT STANDARDS

Below are the standards addressed in this unit.

Readiness Standards	Supporting Standards
G.9(A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems. **G.9(B)** apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean Theorem, including Pythagorean triples, to solve problems	There are no supporting standards in this unit.

Focus on Disciplinary Literacy	
	Mathematical Process Standard **(F)** – analyze mathematical relationships to connect and communicate mathematical ideas
	Mathematical Process Standard **(G)** – display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication

LEARNING SUPPORTS BY LESSON

There is a checkmark for the math support if the lesson	Lessons → / Math Supports	L1	L2	L3	L4	L5	L6	L7	L8
makes a connection to prior content or from a previous unit or academic year	Access Prior Knowledge	✓	✓	✓	✓	✓	✓	✓	✓
uses familiar contexts or experiences to make the learning relevant to students	Real-World Connections							✓	
makes use of graphic organizers	Graphic Organizers			✓					
includes tools like rulers, protractors, patty paper, algebra tiles, etc.	Tools or Manipulatives			✓					
incorporates tables, reference charts, displays, pictures, models, or color-coding	Visual Aids	✓	✓	✓	✓	✓	✓	✓	✓
includes definitions, examples vs. nonexamples, cognates, etc.	Vocabulary Supports								
includes strategies that support language development									
asks students to discuss with their partner to prepare for whole class discussion	- Turn and Talk	✓	✓	✓	✓		✓	✓	✓
teacher facilitates a whole class discussion to debrief key learnings	- Guided Discussion	✓	✓	✓	✓		✓	✓	✓
asks students to think independently, test their idea with a partner, and share whole group	- Think, Pair, Share								
includes sentence stems to support students with explanations	- Sentence Stems								
provides opportunities for students to work with a partner or a group	Peer Collaboration	✓	✓	✓	✓	✓	✓	✓	✓
uses mnemonics such as SohCahToa	Mnemonics								
includes websites or equipment that enhances the lesson	Technological Support								
content can be presented in different forms									
uses hands-on tools or manipulatives to represent the math	- Concrete			✓					
uses drawings to represent the math	- Pictorial	✓	✓	✓	✓	✓	✓	✓	✓
uses numbers and number sentences to represent the math	- Abstract	✓	✓	✓	✓	✓	✓	✓	✓

The EFFL Model

Experience First, Formalize Later (EFFL) Model

Opening

For every new lesson, the teacher begins by making the goals of the lesson crystal clear. The teacher does more than simply read the objective to the class. They make connections to previous learning, share how this learning fits into a bigger picture, or explain why this learning is important for future learning.

Activity / Interaction With New Material (INM)

For this part of the lesson, students work in pairs or groups of four to experience new content through an activity. Students might be discussing a proposed scenario, working with other groups, or doing a simulation. The student activity is designed for students to be able to do without the help of the teacher. Of course, the teacher is watching and listening in to conversations in order to formatively assess student understanding. The teacher provides questions, cues, and prompts (not answers!) to help push groups forward when they are stuck or have made a mistake. As students begin to finish the activity, the teacher identifies students to write their work on the board. Most often, the teacher selects student work that will easily allow them to connect the experience to formal learning. Students write their work on the whiteboard in a single-color marker.

Debrief Activity

Once students have recorded their responses in their workbook (see blue writing to the right), the teacher calls the whole group back together for a debrief. It is in this discussion that the teacher will help students formalize the learning. The teacher connects the student activity experience to new vocabulary, definitions, formulas, and algorithms. The formal learning is attached specifically to the experiences of the activity so that students can enhance their constructed understanding of the new content. The teacher writes all of the formal learning in a different color in the margins of the activity (see red writing to the right). The students add these ideas in the margins on their activity page and often think of this as the formal "notes" of the lesson. In all of the answer keys we provide on Math Medic, the teacher formal learning points are provided in the margins in a different color.

QuickNotes

In this part of the lesson, the teacher uses the whole experience of the activity and the formalization in the debrief to summarize the learning from the lesson. Notice that we use the box to constrain the amount of formal "notes" that the teacher can provide.

Student Practice

Now that students have arrived at some new learning, they need to be able to apply it in new contexts. Most often we have students complete these questions in pairs and occasionally we select one question to use as an exit ticket. If we have time, we have students write solutions on the whiteboard.

Extra Practice

We typically give students around 3-5 "Extra Practice" problems for each lesson. We choose problems that are closely aligned with the Learning Objectives of the lesson. It is our belief that "less is more" here. We would rather students spend their Extra Practice time thinking deeply about just a few problems, rather than surface level thinking on many problems. When possible, we provide the answers at the bottom of the page, so they can immediately assess their understanding.

Slightly modified version of: https://www.calc-medic.com/post/experience-first-formalize-later#:~:text=%E2%80%9CExperience%20First%2C%20Formalize%20Later%E2%80%9D,at%20formal%20definitions%20and%20formulas.

Before You EFFL!

Here are helpful resources that you guide you in the right direction before your first EFFL lesson!

Why Should We EFFL?
The article advocates for the Experience First, Formalize Later (EFFL) teaching model, emphasizing its effectiveness in fostering deep understanding and flexible thinking in students. The author compares traditional teaching to a game of "Simon Says," where students merely mimic instructions without grasping underlying concepts. In contrast, EFFL encourages students to engage actively with problems, enhancing their ability to understand and apply calculus concepts creatively.

Tips for Lesson Planning
The article offers practical advice for effective lesson planning beyond the exhaustive and overly detailed approaches often emphasized during teacher training. It underscores the importance of thoughtful preparation but rejects the notion that teachers need to script every minute or detail of a class session.

Making the Most of Your EFFL Lesson Debrief
The article discusses the significance of the debriefing phase in the Experience First, Formalize Later (EFFL) lesson model, emphasizing its role in reinforcing learning and highlighting student contributions. The debrief session is seen as crucial for integrating academic vocabulary, emphasizing key lesson understandings, and valuing students' mathematical insights.

While You EFFL!

While each lesson may be unique in context and skills, all lessons benefit from the following practices:

Teacher Look Fors:
- ❑ Utilizing the Do Now to spark students' interest in the Activity.
- ❑ Use questioning to promote small group discussion and exploration, guided by monitoring questions.
- ❑ Connects Experience First to formal concepts using a colored pencil/pen to take notes along the margin during the Debrief.
- ❑ Facilitates whole-class discussions for students to reflect, share insights, and provides feedback that reinforces key concepts.
- ❑ Tracks time to adapt lesson pacing and support based on student response and engagement.

Students Look Fors:
- ❑ In the Activity, students engage in group work and discourse.
- ❑ Exploring the activity, testing hypotheses and approaches (trial & error).
- ❑ Take notes on key ideas and concepts using different colored pencil/pen to take notes along the margin.
- ❑ Share thoughts and ideas that demonstrate their approach to their work.

Other considerations
- During the **Experience First** phase, if most of your students seem stuck or disengaged, take a moment to pause, reset, and provide clear instructions. Some problems of the Activity are more suitable to do a whole-class discussion as a means to save some instructional time for Student Practice or the Exit Ticket. You are encouraged to adapt the EFFL (Experience First, Formalize Later) process to meet your students' needs while maintaining a focus on student-centered instruction.

ROADMAP

Day	Date	Lesson	Lesson Title
span AT A GLANCE: Unit 7 – Special Right Triangles and Trigonometry			

Day	Date	Lesson	Lesson Title
There are 4 flexible Success Days that you can use anywhere in the unit.			
• Consider using 1 day to facilitate Lesson 5, a Mid-Unit review that covers Lessons 1 – 4.			
• Consider using 1 day to administer Topic Quiz A between Lessons 5 and 6, and another day to administer Topic Quiz B after Lesson 7.			
• Consider using 1 day to review the day before UE7. For this review, consider using the provided review lesson, Lesson 7.8.			
1		1	45° - 45° - 90° Triangles
2		2	30° - 60° - 90° Triangles
3		3	Trigonometric Ratios
4		4	Trigonometric Ratios: Finding Sides
5		5	Mid-Unit Review A (Lessons 1 – 4)
6			Unit 7 Topic Quiz A	Success Day
7		6	Inverse Trig Ratios: Finding Angles
8		7	Applications of Trigonometry
9			Unit 7 Topic Quiz B	Success Day
10		CR	Cumulative Review	Success Day
11			Unit 7 Exam

Lesson 1: 45° - 45° - 90° Triangles	Date: _____

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

- ◆ **G.9(B)** apply the relationships in special right triangles ~~30°-60°-90°~~ and 45°-45°-90° and the Pythagorean Theorem, ~~including Pythagorean triples,~~ to solve problems

Important Vocabulary
- ▪ **45°-45°-90° triangle**
- ▪ leg
- ▪ hypotenuse
- ▪ **radical**
- ▪ **rationalize**

Notes for Intellectual Preparation & Lesson Planning

<u>Necessary Materials and Pre-Lesson Prep</u>
- ▪ Unit 7 Student Edition
- ▪ Class set of red pens

Lesson Structure:
- 🟥 Do Now (7 min)
- 🟦 INM (17 min)
- 🟩 Debrief (5 min)
- 🟪 Student Practice (16 min)
- 🟪 Exit Ticket (10 min)

<u>Mathematical Goal of this Lesson</u>
By the end of this lesson, students should be able to apply properties of 45° - 45° - 90° triangles to find missing side lengths. Students will need to be able to simplify radicals and rationalize radical expressions, something they learned during Algebra 1 but may not remember. With this in mind, consider using a previous flexible success day to review how to work with radical expressions.

<u>Opportunities to CFU</u>
- ✓ INM: 3, 5
- ✓ Student Practice: 1, 2, 4

<u>Other Notes to Inform Your Planning</u>

For the **Do Now:** The Do Now is not married to the INM, but it does activate students' prior knowledge of the Pythagorean Theorem that they need to be able to reason through INM #1.

On **scaffolding**: Keep in mind that even though the idea of the 45° - 45° - 90° triangle is "new," the idea of triangle similarity is NOT. Whenever students are struggling to make the leap from one question to the next, continually revisit the following ideas that they have seen before: 1) isosceles triangles have two congruent legs; 2) the ratio of the leg to the hypotenuse is always the SAME for similar triangles, because all 45-45-90 triangles are similar by AA~.

Focus on Disciplinary Literacy
INM #3

Lesson Look Fors

<u>Look for teachers to...</u>
- ❑ avoid the temptation to start the lesson by telling students the ratio of the leg to the hypotenuse for all 45° - 45° - 90° triangles. It is important that students come to this realization on their own during INM #s 1-4.
- ❑ build on what students have already learned about similar triangles, since all 45° - 45° - 90° triangles are similar to one another. (I.e., in Unit 6, students were given side lengths and had to determine if two triangles were similar, OR were told a pair of triangles was similar and had to find a missing side. In THIS lesson, students notice patterns in the ratios of sides for all 45° - 45° - 90° triangles).

<u>Look for students to...</u>
- ❑ notice that the two legs of a 45° - 45° - 90° triangle are always equal.
- ❑ notice that the ratio of the hypotenuse to the leg of a 45° - 45° - 90° triangle is always $\sqrt{2}:1$.
- ❑ either be comfortable working with radical expressions on paper OR using the calculator (calculations should not be the focus of this lesson; rather, understanding the ratio of side lengths should be).

Student Know/Do Chart

- 🎯 *Do* — Students can find missing side lengths in a 45°-45°-90° triangle given one side length.
- 🎯 *Do* — Students can simplify radicals and rationalize radical expressions.
- 🎓 *Know* — All 45°-45°-90° triangles are isosceles, so the two legs are always congruent.
- 🎓 *Know* — If you know the length of one leg of a 45°-45°-90° triangle, you can get the hypotenuse by multiplying that leg by $\sqrt{2}$.
- 🎓 *Know* — IF you know the hypotenuse of 45°-45°-90° triangle, you can get the leg by dividing the length of the hypotenuse by $\sqrt{2}$.

Lesson 2: 30° - 60° - 90° Triangles		Date: _____
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **G.9(B)** apply the relationships in special right triangles 30°-60°-90° ~~and 45°-45°-90°~~ and the Pythagorean Theorem, ~~including Pythagorean triples,~~ to solve problems

Important Vocabulary
- **30°-60°-90° triangle**
- leg
- hypotenuse
- radical
- rationalize

Notes for Intellectual Preparation & Lesson Planning

<u>Necessary Materials and Pre-Lesson Prep</u>
- Unit 7 Student Edition
- Class set of red pens
- Class set of protractors and rulers

Lesson Structure:
- 🟥 Do Now (7 min)
- 🟦 INM (18 min)
- 🟩 Debrief (3 min)
- 🟪 Student Practice (17 min)
- 🩷 Exit Ticket (10 min)

<u>Mathematical Goal of this Lesson</u>
By the end of this lesson, students should be able to apply properties of 45° - 45° - 90° triangles to find missing side lengths. As in the previous lesson, students will need to be able to simplify radicals and rationalize radical expressions. Just like in lesson 1, students should realize that all 30°-60°-90° triangles are similar; consequently, the ratio of side lengths is always the same.

<u>Opportunities to CFU</u>
✓ INM: 2, 5, 8   ✓ Student Practice: 1, 2, 3

<u>Other Notes to Inform Your Planning</u>
For the **Do Now:** The Do Now spirals in Lesson 1. It can be skipped or replaced; however, it does serve to activate students' prior knowledge for the INM.

For **INM:** Each student will have created a different triangle in INM#1 based on wherever they plotted point A. This is intentional – the goal is for students to see that regardless of size, the hypotenuse is always double the length of the short leg for all 30-60-90 triangles.

┌─────────────────────────┐
│ **Focus on**   │
│ **Disciplinary Literacy** │
│   │
│   INM #6   │
└─────────────────────────┘

On **scaffolding**: Each INM question builds upon the previous one. #s 1-6 get students to notice that the hypotenuse is always twice the length of the short leg in a 30-60-90 triangle. In #7, students use this generalization to find the short leg, and then use the Pythagorean Theorem to find the long leg. In #8 they are able to notice that the long leg is always $\sqrt{3}$ times bigger than the short leg, allowing them to see that the ratio of a 30°-60°-90° triangle's side lengths from least to greatest is always $x : x\sqrt{3} : 2x$

Lesson Look Fors

<u>Look for teachers to...</u>
☐ avoid the temptation to start the lesson by telling students the ratio of side lengths for all 30° - 60° - 90° triangles. It is important that students come to this realization on their own during the INM.
☐ build on what students have already learned about similar triangles, since all 30° - 60° - 90° triangles are similar to one another. (I.e., in Unit 6, students were given side lengths and had to determine if two triangles were similar, OR were told a pair of triangles was similar and had to find a missing side. In THIS lesson, students notice patterns in the ratios of side lengths.)

<u>Look for students to...</u>
☐ notice that the hypotenuse is always double the length of the short leg in a 30° - 60° - 90° triangle.
☐ notice that the longer leg is always $\sqrt{3}$ greater than the short leg in a 30° - 60° - 90° triangle.

Student Know/Do Chart

🎯 **Do** Students can find missing side lengths in a 30°-60°-90° triangle given one side length.

🎯 **Do** Students can simplify radicals and rationalize radical expressions.

🎓 **Know** All 30°-60°-90° triangles are similar; the ratio of their side lengths from least to greatest is $x : x\sqrt{3} : 2x$

🎓 **Know** If you know the length of one leg of a 30°-60°-90° triangle, you can use the ratio above to find the other two legs.

Lesson 3: Trigonometric Ratios	Date: _____

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)

◆ **G.9(A)** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems.

Important Vocabulary
- **cosine**
- **sine**
- **tangent**
- **trigonometry**
- **adjacent**
- **opposite**
- **reference angle**

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- Unit 7 Student Edition
- Class set of pre-cut pipe cleaners (see p2 for details)
- Class set of red pens
- Class set of rulers

Lesson Structure:
- 🟥 Do Now (7 min)
- 🟦 INM (12 min)
- 🟩 Debrief (10 min)
- 🟪 Student Practice (16 min)
- 🟪 Exit Ticket (10 min)

Mathematical Goal of this Lesson
By the end of this lesson, students should be able to write sine, cosine, and tangent ratios for both acute angles in a right triangle and interpret their values. The focus of this lesson is not on memorizing SohCahToa (although it is a helpful mnemonic), but on coming to the realization that "sine" "cosine," and "tangent" are meaningful ratios because all similar triangles have the same angle measures and ratio of side lengths.

Opportunities to CFU
- ✓ INM: 11, 12
- ✓ Student Practice: 1, 2, 3

Other Notes to Inform Your Planning

For the **Do Now:** The Do Now spirals in previous content. It can be skipped or replaced if necessary; however, it does activate students' prior knowledge of special right triangles, which connects to the idea that all similar triangles have the same ratio of side lengths, which is true even for triangles beyond 45-45-90 or 30-60-90.

For **INM**: You will need pre-cut pipe cleaners for this activity. Please carefully read TE p27 for details.

On **scaffolding**: Each INM question builds upon the previous one. #s 1-6 guide students through creating a right triangle with their pipe cleaners, recording the ratio of side lengths, and sharing their data with the class. Question 8-12 help students see that similar triangles have similar ratios of side lengths. The Debrief (all the red ink, TE pp 29-30) is where students formalize what they have just experienced – it's where you introduce sine, cosine, tangent, and stamp the fact that similar triangles have identical sin/cos/tan ratios because they have congruent angles.

Focus on Disciplinary Literacy

Debrief

Lesson Look Fors

Look for teachers to…
- ❑ avoid use of the calculator (except to turn fractions to decimals if students struggle with that), since students do not need to find missing side lengths. Students are simply identifying appropriate ratios based on given side lengths of a triangle.
- ❑ during the QuickNotes, stamp that each ratio of a right triangle has a specific name and that its value is determined by the angle measure; present SohCahToa as a helpful mnemonic.

Look for students to…
- ❑ realize that "sine," "cosine," and "tangent" are ratios, and that similar triangles will have identical values for these ratios that are determined by the angle measure.

Student Know/Do Chart

Do 🎯 Students can select a triangle from a set of triangles for which $\sin \theta = \frac{4}{5}$ is true.

Know 🎓 "Sine" is the trigonometric function that is equal to the ratio of the side opposite a given angle (in a right triangle) to the hypotenuse.

Lesson 4: Trigonometric Ratios: Finding Sides		Date: _____
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **G.9(A)** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems.	**Necessary Materials and Pre-Lesson Prep** ▪ Unit 7 Student Edition ▪ Class set of red pens **Lesson Structure:** 🟥 Do Now (7 min) 🟦 INM (13 min) 🟩 Debrief (10 min) 🟪 Student Practice (15 min) 🟪 Exit Ticket (10 min) **Mathematical Goal of this Lesson** By the end of this lesson, students should be able to use trigonometric ratios to find side lengths of right triangles. This lesson makes an explicit connection between similar triangles and right triangle trigonometry. Because similar triangles will have the same angle measures and ratio relationships between their sides (just as the pattern for 45-45-90 triangles is $x:x:x\sqrt{2}$), if you know the ratio relationship (sine or cosine or tangent) and one acute angle of a right triangle, you can find all other side lengths of that triangle.	**Look for teachers to...** ❑ (when debriefing the Do Now) emphasize the ratio relationship from the leg to the hypotenuse (1:3) as opposed to the ratio between corresponding sides (6:10). ❑ do not use sin/cos/tan on the calculator until you debrief #6; students do not need it until then. **Look for students to...** ❑ realize that trig ratios are the key to unlocking any right triangle because it gives the same information that having a similar triangle would. ❑ think of their calculator as a way to "look up" the ratio for any acute angle in a right triangle (see TE pp 35-36 for more details).
	Opportunities to CFU ✓ INM: 2, 5, 6 ✓ Student Practice: 1, 2, 3	**Student Know/Do Chart**
Important Vocabulary ▪ cosine ▪ sine ▪ tangent ▪ trigonometry ▪ adjacent ▪ opposite ▪ reference angle	**Other Notes to Inform Your Planning** For the **Do Now:** The Do Now sets up the INM. It cannot be skipped or replaced. On **scaffolding**: Each question in the Do Now and INM build upon each other. In Q1-Q2, students should realize that the hypotenuse is three times the given side length for BOTH triangles, which makes sense because they're similar. In Q3, students connect that previous realization to the sine ratio to realize the sine of the same reference angle is $\frac{1}{3}$ in both cases. In Q4, students are given the sine, and they can use it to set up a proportion to find a missing side length. When discussing Qs 4-5, be sure to ask "Is this triangle similar to the ones we saw in the Do Now? How do you know?" **Focus on Disciplinary Literacy** INM #5	**Do** Students can apply their knowledge of trigonometric ratios to find missing side lengths. **Know** When you want to know the adjacent side length and are given the hypotenuse, use cos⁻¹. **Know** When you want to know the opposite side length and are given the adjacent, use tan⁻¹.

Lesson 5: Mid-Unit Review A (Lessons 1-4) – **RALLY/COACH!!!**		Date: _____
Standard(s)	**Notes for Intellectual Preparation & Lesson Planning**	**Lesson Look Fors**

Standard(s)

◆ **G.9(A)** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems.

◆ **G.9(B)** apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean Theorem, including Pythagorean triples, to solve problems

Important Vocabulary

All vocabulary from Lessons 1 – 6.

Notes for Intellectual Preparation & Lesson Planning

Necessary Materials and Pre-Lesson Prep
- Unit 7 Student Edition
- Decide how to pair students

Lesson Structure:
- 🟥 Do Now (7 min)
- 🟦 INM (40 min)
- 🟩 Debrief (8 min)
- 🟪 Student Practice (0 min)
- 🟪 Exit Ticket (0 min)

Mathematical Goal of this Lesson
This lesson provides a review of Lesson 1-4 and gives students the opportunity to give and receive feedback to a partner.

Opportunities to CFU
✓ when giving Rally/Coach directions (before you release students to participate in the activity)
✓ during the Debrief when you go over most-missed questions

Other Notes to Inform Your Planning

For the **Do Now:** The provided Do Now is helpful for activating prior knowledge, but it is not married to the INM and can be replaced with a spiraled review question of your choice.

For **Debrief:** Go over questions students struggled with the most.

For the **Exit Ticket**: No SP is provided because this review lesson is essentially a LOT of student practice. No ET is provided because the next day should be a Topic Quiz day that will give you a great data point.

Focus on Disciplinary Literacy

Rally / Coach

Lesson Look Fors

Look for teachers to…
❑ give clear directions on "Rally/Coach" protocol so the activity can flow smoothly.
❑ circulate and monitor, listening in for problems that stump students, and go over these questions before students leave so they get a sense of closure.

Look for students to…
❑ talk with their partners about the math problem they're working on at the time.
❑ persevere through challenge. If neither partner knows how to solve a problem, that is okay! They can try a different problem and come back to it later. If they're totally stuck, they can ask for help.

Student Know/Do Chart

Do 🎯 Students can apply all "Dos" from Lessons 1-4.

Know 🎓 Students know all "Knows" from Lessons 1-4.

Lesson 6: Inverse Trigonometric Ratios: Finding Angles		Date: _____
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **G.9(A)** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems. ◆ **G.9(B)** apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean Theorem, including Pythagorean triples, to solve problems	**Necessary Materials and Pre-Lesson Prep** ▪ Unit 7 Student Edition ▪ Class set of red pens Lesson Structure: 🟥 Do Now (7 min) 🟦 INM (12 min) 🟩 Debrief (10 min) 🟪 Student Practice (16 min) 🟪 Exit Ticket (10 min) **Mathematical Goal of this Lesson** By the end of this lesson, students should be able to use trigonometric ratios to find **angle** measures in right triangles. From Unit 6, students know that if the sides of a triangle are proportional, then the triangles are similar, and thus the angles are congruent. Today we expand this slightly to say that if we have a particular ratio of sides (given by the sine, cosine, or tangent) then only one type of triangle is possible and the angles are locked in. We can "look up" the angles that go with a particular ratio using the inverse trig functions, just like we can "look up" the ratios that go with a particular acute angle in a right triangle. **Opportunities to CFU** ✓ INM: 4, 5, 7 ✓ Student Practice: 1, 2, 3 **Other Notes to Inform Your Planning** For the **Do Now:** The Do Now sets up the INM and cannot be skipped or replaced. For **Debrief**: Entering precise mathematical language in the calculator is part of disciplinary literacy. Use TI Nspire software or show the steps you take on your calculator under the doc cam. On **scaffolding**: please see scaffolding questions provided on TE p51.	Look for teachers to... ❑ model how to use the calculator to find missing angles during the debrief, NOT during the INM. ❑ actively monitor during the INM, asking scaffolding questions provided on TE p51 when students get stuck. Look for students to... ❑ notice that the triangles in #s 1-6 are similar, and use this relationship to identify angle measures and side lengths without the calculator. ❑ be able to explain why, in #7, $m\angle J \neq m\angle X$ (the triangle in #7 is not similar to the triangles in #s 1-6).

Important Vocabulary
▪ cosine ▪ **inverse cosine** ▪ **inverse sine** ▪ **inverse tangent** ▪ sine ▪ tangent

Focus on Disciplinary Literacy

DEBRIEF

Student Know/Do Chart

Do 🎯 Students can apply their understanding of inverse trigonometric ratios to find missing angles.

Do 🎯 Students can use their calculator as a reference tool to "look up" angle measures given a right triangle's side lengths.

Know 🎓 Students know what series of buttons to press on the calculator that will allow them to input side lengths and output an angle measure.

Know 🎓 When given the adjacent side and hypotenuse for a missing angle, use inverse cosine. When given the opposite and adjacent side, use inverse tangent.

Lesson 7: Applications of Trigonometry		Date: _____
Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **G.9(A)** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems.	<u>Necessary Materials and Pre-Lesson Prep</u> ▪ Unit 7 Student Edition ▪ Class set of red pens ▪ Plane crash scene (from Sully) **Lesson Structure:** 🟥 Do Now (7 min) 🟦 INM (15 min) 🟩 Debrief (7 min) 🟪 Student Practice (16 min) 🟪 Exit Ticket (10 min) <u>Mathematical Goal of this Lesson</u> By the end of this lesson, students should be able to solve right triangles in application problems involving the Pythagorean Theorem and trig ratios. While students already know how to solve for missing sides and angles in a right triangle, the additional skill today is to interpret text and be able to draw and label a diagram that represents known info and use it to solve. <u>Opportunities to CFU</u> ✓ INM: 2, 3, 4 ✓ Student Practice: 1, 2, 3	<u>Look for teachers to…</u> ❑ read the "Miracle on the Hudson" box and show the clip from *Sully* before releasing students to work on the INM. ❑ get students excited about this very recent, very real application of mathematical reasoning that saved lives. ❑ as students struggle through the INM, circulate and offer the minimum required support possible when students are too stuck to move on. <u>Look for students to…</u> ❑ persevere through the challenge ❑ annotate the diagram provided in INM#2 with information as they figure it out to help them "see" the triangles they didn't notice before.
Important Vocabulary ▪ **angle of depression** ▪ **angle of elevation** ▪ cosine ▪ inverse cosine ▪ inverse sine ▪ inverse tangent ▪ sine ▪ tangent	<u>Other Notes to Inform Your Planning</u> For the **Do Now:** The Do Now is part of the lesson! Ensure you can show the video clip. There's no computation required for the Do Now – just student discussion that helps set the stage for today's INM. On **scaffolding**: Students must answer the questions in order, as each one provides information that is useful to the next. When students are stuck, ask, "What are we trying to find out?" and then, "What information do we already have that will help us get the information we need?" If students are still stumped, you can ask, "Does this look like problems we've done the last few days?" If that doesn't work, try "Can we set up a trig ratio with any of this information?" For example, in #2, we need to find the plane's height above the ground when it hits the birds (labeled *x* in exemplar on TE p66). So, we need to find the side OPPOSITE the angle 7.7°, and we're given the hypotenuse of 4. We can use the sine ratio to find *x*! (Note that annotating the diagram or drawing a sketch is crucial.) **Focus on Disciplinary Literacy** Debrief of #2	<u>Student Know/Do Chart</u> Do 🎯 Students can interpret text to solve a right triangle application problem (which for most students also involves drawing and label a diagram that represents known information). Do 🎯 Students can use their calculator to solve trigonometry problems. Know 🎓 The "angle of depression" is the angle between the horizontal line (what pilot would see looking straight ahead) and the observation of the object from the horizontal line (the "line of sight" from pilot's eyes to object on the ground). Know 🎓 In the context of the problem, we are given the adjacent side and need to find the opposite side, so we need to use the tangent ratio.

Cumulative Review	Success Day	Date: _____
Standard(s)	**Notes for Intellectual Preparation & Lesson Planning**	**Lesson Look Fors**

Standard(s)	Notes for Intellectual Preparation & Lesson Planning	Lesson Look Fors
◆ **G.9(A)** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems. ◆ **G.9(B)** apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean Theorem, including Pythagorean triples, to solve problems	**Necessary Materials and Pre-Lesson Prep** ▪ Review students' Unit 7 exit ticket data to determine what to prioritize during review ▪ Internalize Review Lesson 7.8 if you choose to use it **Lesson Structure:** 🟥 Do Now (7 min) 🟦 INM (38 min) 🟩 Debrief (10 min) 🟪 Student Practice (0 min) 🟪 Exit Ticket (0 min) **Mathematical Goal of this Lesson** By the end of this class period, students should get the opportunity to review major concepts from Unit 7. **Other Notes to Inform Your Planning** You should use this Success Day to review however you see fit. An optional review activity has been provided in the Teacher Edition called "Around the World" (Review Lesson 8.7). To prepare for today's Around the World review activity, print the questions and cut them up into individual cards. Distribute them around the room (they don't have to be alphabetical). Refer students to the recording sheet in their Student Workbook and have them travel "around the world" with a partner to complete the problems, keeping track of their work in the appropriate box. These problems require students to demonstrate conceptual understanding and procedural skill. Questions B, G, and K all have students think about the trig ratios of comparisons of sides. Students should be able to explain why the sine and cosine can't be greater than 1 without being told this explicitly or knowing the graphs of the functions (which they will do in Algebra 2 and Precalculus). They also should be able to interpret a ratio of 1 as representing a 1:1 ratio, as in the legs of an isosceles right triangle. Additionally, students will see that while it is not possible for the sine or cosine ratios to be greater than or equal to 1, the tangent ratio can exceed 1. Question L reinforces the fact that many triangles can have the same ratio of sides, since they can be scaled versions of each other, and these triangles are geometrically similar. (See TE p71 for more details.)	**Look for teachers to...** ❑ facilitate a review session that prioritizes what students need based on previous exit ticket and/or quiz data. **Look for students to...** ❑ review Unit 7 topics in preparation for the Unit 7 exam. **Student Know/Do Chart** Do🎯 Students can prepare for the Unit 7 Exam by reviewing Unit 7 content.

Recommended Unit 7 <u>Success Day</u> Material and Resources

For more practice with 45°-45°-90° and 30°-60°-90° triangles (Lessons 1-2), try...
- 22-23 MCR Lesson 6.3: 45°-45°-90° Triangles: SW | TE
- 22-23 MCR Lesson 6.4: 30°-60°-90° Triangles: SW | TE
- 22-23 MCR Lesson 6.5: Applications of Special Right Triangles: SW | TE
- Kahoot: 45°-45°-90° Triangles
- Kahoot: 30°-60°-90° Triangles
- Kuta: Special Right Triangles
- GeoGebra: Special Right Triangles
- ThatQuiz: 30-60-90 Triangles

For more practice with trigonometric ratios and inverse trigonometric ratios (Lessons 3-6), try...
- 22-23 MCR Lesson 6.6: Trig Ratios: Finding Sides Day 1: SW | TE
- 22-23 MCR Lesson 6.7: Trig Ratios: Finding Sides Day 2: SW | TE
- 22-23 MCR Lesson 6.8: Trig Ratios: Finding Angles Day 1: SW | TE
- 22-23 MCR Lesson 6.9: Trig Ratios: Finding Angles Day 1: SW | TE
- Imagine Math: Relate Sides and Angles of a Triangle: SW | TE
- Imagine Math: Use Trigonometric Ratios to Find Angles: SW | TE
- Imagine Math: Use Trigonometric Ratios to Find Sides: SW | TE
- General Trigonometry Practice
- GeoGebra: Identifying Trig Ratios: Quick Formative Assessment
- GeoGebra: Right Triangles: Identifying Sides

For more practice with applications of solving right triangles, (Lesson 7), try...
- 22-23 MCR Lesson 6.10: Trig Ratios: Angle of Elevation and Depression Day 1: SW | TE
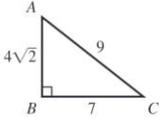- 22-23 MCR Lesson 6.11: Trig Ratios: Angle of Elevation and Depression Day 2: SW | TE
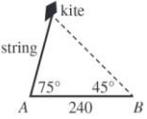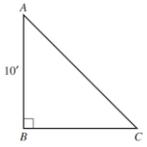- Kahoot: Angles of Elevation and Depression

For General Review, try...
- Progressive Math Initiative: Similar Triangles & Trigonometry Problems

Unit 7 Exam	Date: _____
Standard(s)	Notes for Intellectual Preparation & Lesson Planning
◆ **G.9(A)** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems. ◆ **G.9(B)** apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean Theorem, including Pythagorean triples, to solve problems	**Necessary Materials and Pre-Lesson Prep** ▪ Print enough copies of UE7 Bluebook on Curriculum Corner (for the FRQ) and ensure you have access to the exam on Edcite. (If you do not have access to the exam, notify your TC and submit a Zendesk ticket ASAP.) **Notes to Inform Your Planning** Review the Unit 7 Exam on Curriculum Corner. Internalize and create an exemplar for the assessment prior to teaching the unit as part of unpacking the unit. Use your exemplar to spar with the solutions provided in the Assessment Companion on Curriculum Corner. The scanning deadline for the Unit 7 Exam is **February 19th, 2026**. Consider administering the exam 1-3 school days BEFORE February 19th to allow sufficient time for grading the FRQ. Refer to the scoring guide to score the FRQ.

UNPACKED STANDARDS

Focus standards for this unit.

Standards Clarification		
Standards	**Specificity**	**Notes/Explanations/Examples**
G.9A determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems	Content: • Sine, cosine, tangent • Inverse trigonometric functions • Law of Cosines and Law of Sines • Angles of Elevation and Depression Including but not limited to: • Using sine, cosine, and/or tangent to find missing sides. • Using inverse sine, inverse cosine, and/or inverse tangent to find missing angles. • Using the angles of elevation and/or depression to find the distance between two objects • Using the Law of Sines to solve triangles.	ACT: Click on image to access source. 17. In right triangle $\triangle ABC$ shown below, the given lengths are in millimeters. What is sin A? A. $\frac{4\sqrt{2}}{9}$ B. $\frac{4\sqrt{2}}{7}$ C. $\frac{7\sqrt{2}}{8}$ D. $\frac{7}{9}$ E. $\frac{9}{7}$ 59. The figure below shows a flying kite. At a certain moment, the kite string forms an angle of elevation of 75° from point A on the ground. At the same moment, the angle of elevation of the kite at point B, 240 ft from A on level ground, is 45°. What is the length, in feet, of the string? A. $60\sqrt{3}$ B. $80\sqrt{6}$ C. 144 D. 180 E. 240
G.9B apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems	Content: • Special Right Triangles (30-60-90 and 45-45-90) • Pythagorean Theorem and its converse • Pythagorean Triples Including but not limited to: • Using the Pythagorean Theorem to find missing sides. • Using the converse of the Pythagorean Theorem to determine whether a triangle is right or not. • Determining whether three side lengths form a right triangle. • Find missing side lengths of a 45-45-90 triangle. • Find missing side lengths of a 30-60-90 triangle.	ACT: Click on image to access source. 13. In the isosceles right triangle below, AB = 10 feet. What is the length, in feet, of $\overline{AC}$? A. 5 B. 10 C. 20 D. $\sqrt{20}$ E. $10\sqrt{2}$ 34. A surveyor took and recorded the measurements shown in the figure below. If the surveyor wants to use these 3 measurements to calculate the length of the pond, which of the following would be the most directly applicable? F. The Pythagorean theorem G. A formula for the area of a triangle H. The ratios for the side lengths of 30°-60°-90° triangles J. The ratios for the side lengths of 45°-45°-90° triangles K. The law of cosines: For any $\triangle ABC$, where a is the length of the side opposite $\angle A$, b is the length of the side opposite $\angle B$, and c is the length of the side opposite $\angle C$, $a^2 = b^2 + c^2 - 2bc \cos(\angle A)$
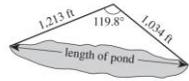
VERTICAL STANDARDS

This section details the **progression** of key student expectations/standards** in the courses **before** and **after** this course. This will help you understand what **prior knowledge skills to build upon** and guide you in knowing what **skills you are preparing your students** for in the subsequent course.

7th Grade	Geometry	Algebra II
7.4A represent constant rates of change in mathematical and real world problems given pictorial, tabular, verbal, numeric, graphical, and algebraic representations, including d = rt. **7.4D** solve problems involving ratios, rates, and percents, including multi-step problems involving percent increase and percent decrease, and financial literacy problems **7.11C** write and solve equations using geometry concepts, including the sum of the angles in a triangle, and angle relationships.	**G.9A** determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems. **G.9B** apply the relationships in special right triangles 30°-60°-90° and 45°-45°-90° and the Pythagorean theorem, including Pythagorean triples, to solve problems.	**2A.6L** formulate and solve equations involving inverse variation.

Algebra I		Pre-Calculus
A.2D write and solve equations involving direct variation.		**P.2F** graph exponential, logarithmic, rational, polynomial, power, trigonometric, inverse trigonometric, and piecewise defined functions, including step functions. **P.5M** use trigonometric identities such as reciprocal, quotient, Pythagorean, cofunctions, even/odd, and sum and difference identities for cosine and sine to simplify trigonometric expressions.